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# When is a tempo effect a tempo distortion?

The measurement of the quantum and tempo of life cycle events is one of the oldest and most important topics in demography. Quantum for a cohort is defined as the average number of events over the course of the life cycle. Tempo for a cohort is defined as some measure of central tendency, usually the mean, of the distribution of ages at which events occur. For example, completed cohort fertility is a quantum measure, and life expectancy at birth in a cohort life table is a tempo measure of mortality.

There are three problems with cohort measures of quantum and tempo. First, they do not provide information on behavior during specific years or other short time period, which is often what we are most interested in. Second, cohort measures can be calculated only for cohorts whose life cycle event experience is complete. This means that cohort measures necessarily refer to behavior in the more or less distant past, sometimes many decades in the past. Third, the calculation of cohort measures requires data for all years in which life cycle events to the cohort occur. Often this data is unavailable, even for fertility and nuptiality, which require roughly three decades of data. In the case of mortality, data is required for as much as 100 years, a severe limitation.

"Period" measures overcome these problems. Familiar cohort measures, such as the average number of births per woman, mean age at childbearing, and expectation of life at birth, can be calculated from age specific event rates observed over the cohort's life cycle. Period versions of these measures are defined as t*he value of the measure in a hypothetical cohort that experiences, throughout its lifetime, the rates observed in the reference time period.*

Period measures are very widely used, even though it has been known for more than half a century that there are situations in which such measures do not adequately represent current conditions in the population. For example, Ryder (1956) observed that the period total fertility rate is distorted when women advance or postpone births. In his writing on this topic Ryder often used the terms tempo effect and tempo distortion to refer to the divergence of period indicators from corresponding cohort parameters. In our earlier work

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on period tempo effects we have adopted Ryder's terminology, but the practice of assuming all tempo effects to be distortions is now being questioned. For example, Rodrigues (2006) has argued, that in the case of mortality, there are tempo effects but that these are not distortions. Guillot (2006) and NiBhrolchain (2007) similarly argue that tempo adjusted measures may be appropriate for some purposes but not others. These views have led us to write the present discussion note which aims to clarify the conceptual difference between a tempo *effect* and a tempo *distortion* and to answer the question of when tempo effects should be regarded as distortions.

## 1. BACKGROUND

The terms "tempo effect" and "tempo distortion" were first introduced in the demographic literature by Norman Ryder, who made a series of fundamental contributions to the study of quantum and tempo measures in fertility (Ryder, 1956, 1959, 1964, 1980). His most important finding was that a change in the timing of childbearing of cohorts results in a discrepancy between the period total fertility rate (*TFR*) which measures the fertility of a hypothetical cohort and the completed fertility rate (*CFR*) which measures the fertility of an actual cohort. He considered the period *TFR* to contain a tempo distortion when the timing of childbearing changed.

Ryder's work was highly influential and for most of the last half century the idea of tempo effects in fertility has been widely accepted. Empirical assessment of tempo effects, however, was problematic and rarely attempted. This changed in 1998, when we provided a reformulation that lead to a simple equation for assessing period tempo effects that makes relatively modest demands on data (Bongaarts and Feeney, 1998). Our analysis led to considerable discussion and controversy over approaches to measuring fertility tempo effects (Bongaarts, 2002; Bongaarts and Feeney, 2000, 2006; Lesthaeghe and Willems, 1998; Ní Bhrolcháín, 2007, 2008; Van Imhoff and Keilman, 2000; Kohler and Ortega, 2002, 2004; Kohler and Philipov, 2001; Kim and Schoen, 2000; Schoen, 2004; Sobotka, 2003, 2004; Sobotka and Lutz, 2009; Zeng and Land, 2001, 2002).

Further controversy ensued when we proposed that period mortality measures – in particular the conventional life expectancy – are also affected by tempo effects. A substantial, highly technical literature surrounding this issue has accumulated in recent years, much of it summarized in Barbi, Bongaarts and Vaupel (2008).

## 2. TEMPO EFFECTS

If age-specific birth rates are constant for a sufficiently long period of time, both period total fertility rates and cohort total fertility rates will be constant and have the same value. One might infer from this that period total fertility rates and cohort total fertility rates will be equal if both of these rates are constant for a sufficiently long period of time. The latter proposition is false, as shown by Ryder. The period total fertility rate and the cohort fertility rate can be constant for any length of time, while at the same time be different from each other, if the mean age at childbearing is changing. Ryder referred to this difference between the period and cohort indicators as a "tempo" effect.

To illustrate this point, see Figure 1 in which the dots represent the density and distribution of some demographic event by period (vertical lines) and cohort (diagonals). The sum of the age specific rates in a given year gives the period "Total Event Rate," or *TER*. The sum of rates over the life cycle of a cohort gives the cohort total event rate, which we denote as *CER*. If the distribution of events is the same in each year, the period and cohort quantum are constant and equal, *TER*=*CER*.

Figure 1 – *Density of events by period and cohort, constant tempo*

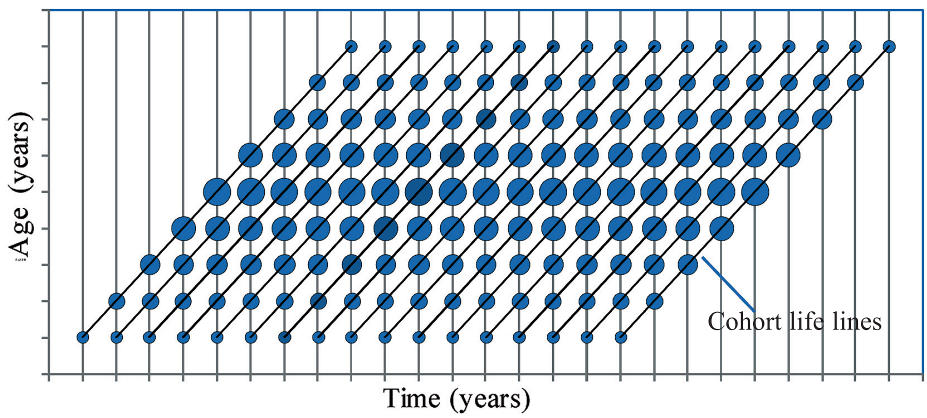
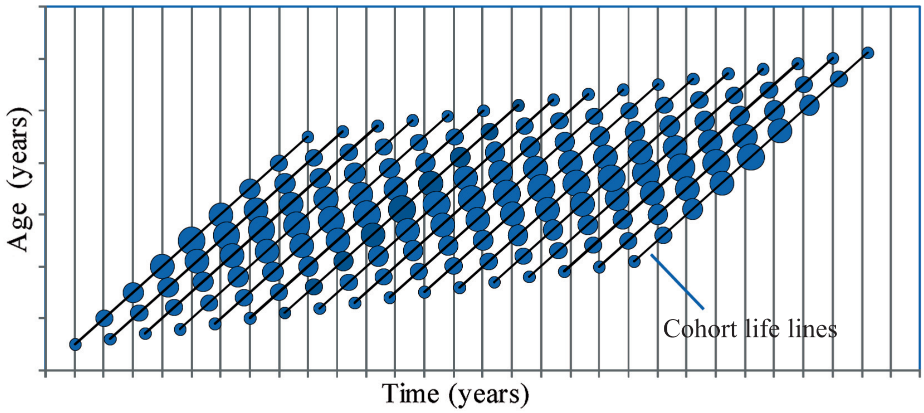



Figure 2 introduces tempo changes by shifting the age distributions of events in cohorts by a fixed amount r each year holding the shape constant. In this scenario the *TER*, *CER* and r are all constant but *TER*<*CER* when $r$>0 (as shown) and *TER*>*CER* when $r$<0 (not shown). A rising mean age at the event deflates the period indicator relative to the cohort level. A declining mean age has the opposite effect. As suggested by the general terminology used in this illustration, tempo effects are not limited to fertility; they may occur for any demographic event.

Figure 2 – *Density of events by period and cohort, increasing tempo*




This brief examination of cohort and period indicators clearly documents the existence of tempo effects, but it does not provide a direct way to measure this effect unless cohort and period fertility are constant, which is generally not the case. We addressed this issue in 1998 when we introduced a procedure for estimating the tempo effect in a period as the difference between (a) the observed value of a period measure at time *t* and (b) the value that would have been observed if the period mean age at the event at time *t* had been constant (Bongaarts and Feeney 1998, 2003, 2006). A *TFR* tempo effect, for example, is the difference between the observed *TFR* for a given year and the value that would have been observed if the period mean age at childbearing had remained constant at the value observed at the beginning of the year. The latter measure is called the tempo adjusted *TFR*. For demographic processes more generally, a tempo effect is defined as the difference between the observed and tempo adjusted *TER*. Tempo effects result from an inflation or deflation of the number of events observed in a period when the period mean age changes[1] (Bongaarts and Feeney, 2006: 35). Note that there is no reference to any cohort measures in this definition. However, the tempo adjusted *TFR* equals the *CFR* of the cohort born in year *t* if the mean age a childbearing remains constant in future years and no other changes in fertility behavior occur[2].

[1] The period mean age at childbearing is cohort size adjusted which means that the mean is derived from observed period age specific birth rates rather than from the number of births by age. The simple formula to estimate the tempo effect is based on the assumption that the shape of the schedule of age-specific fertility rates remains invariant (but the schedule can shift to higher or lower ages and inflate or deflate over time).
[2] Statement is conditional on the tempo-adjusted *TFR* reflecting properly the tempo effect and the assumptions behind it not being violated.

The concept of delayed childbearing for cohorts is straightforward, but a period delay in childbearing is less clear-cut. To clarify our analysis of the impact of period delays in demographic events on period event rates it is useful to distinguish three types of period delay (for simplicity we will use birth delays, but the following discussion applies also to other demographic events):

*Temporary delay*: Births may be delayed in certain years on account of war, economic conditions or cultural beliefs (e.g. Japan's fire-horse year), without any long term trend in the age at childbearing. For example, suppose that postponement occurs only during one year *Y* and that all postponed births occur in the following year. Let *B* denote the number of births that occur each year in the absence of postponement and x the number of births postponed from year *Y* to year *Y*+1. In this scenario the annual number of births will decline from *B* to *B*-*x* between year *Y*-1 and year *Y*, increase from *B*-*x* to *B*+*x* between year *Y* and year *Y*+1, and decline from *B*+x to *B* between year *Y*+1 and year *Y*+2. Period fertility as measured by age-specific birth rates will accordingly fall between years *Y*-1 and Y, rise between years *Y* and *Y*+1, fall again between years *Y*+1 and *Y+2*, and remain constant thereafter. The postponement of births from year *Y* to year *Y*+1 is accompanied by a rise in the mean age at childbearing between year *Y* and year *Y*+1 and a fall between year *Y*+1 and year *Y*+2. These changes result in a negative tempo effect in year *Y* and an offsetting positive tempo effect in year *Y*+1.

*A permanent shift* occurs when period changes in conditions, such as increases in women's education and labor force participation, result in a permanent rise in the age at childbearing. Suppose that the period mean age at childbearing rises during year *Y* because of a change in such conditions[3]. The number of births will decline from *B* to *B-x* between year *Y*-1 and year *Y*, as before. Between year *Y* and year *Y*+1, however, births will rise from *B-x* to *B* (and not to *B+x*) and thereafter will remain constant. There is a one-time negative tempo effect during year *Y*. In this scenario *x* births seem to have disappeared. This is the case from a period perspective, but not from a cohort perspective because the delay has no effect on cohort fertility.

*Continuous rising mean age*. This scenario occurs when the one time permanent shift described in the preceding case is repeated year after year. Instead of a deficit in births in one year the deficit occurs year after year as long as the mean age at childbearing keeps rising. This is equivalent to the scenario depicted in Figure 2 with a negative tempo effect that persists year after year.

[3] The schedule of age specific rates is assumed to maintain its shape; see note 1.

Though very different, these three types of delay render birth rates problematic as indicators of current fertility conditions. Similar problems arise in period measures of other demographic events. Tempo effects in period nuptiality measures have been demonstrated by Winkler-Dworak and Engelhardt (2004). Bongaarts and Feeney (2003, 2003, 2006), Guillot (2006), Luy (2006), Luy and Wegner (2009), Rodriguez (2006). Vaupel (2002, 2005) and Wachter (2006) discuss tempo effects for mortality.

## 3. TEMPO DISTORTIONS

Whether a tempo effect in an indicator is a distortion that requires an adjustment depends on the use to which the indicator is put. Guillot (2006; see also Vaupel, 2002) notes that period indicators have three distinct uses in demography:

(1) To summarize period age-specific rates. The results may be given a synthetic cohort interpretation, but the aim is purely descriptive.

(2) As proxies for actual cohort indicators where the data necessary for the calculation of cohort indicators is not available, e.g., because some of the cohort experience lies in the future.

(3) As estimates of the fertility implied by the continuation (without change) of *current conditions* which are defined as "all underlying factors affecting demographic behavior" at a given point in time. In a hypothetical scenario in which current conditions stay constant in the future period indicators would stabilize at a level Guillot refers to as "stationary-equivalent" or "under current conditions". These terms are consistent with the BF term "tempo adjusted"

If current conditions are fully represented by age-specific birth rates, (1) and (3) are the same. If current conditions are not fully represented by age-specific birth rates, as we argue below, continuation of current conditions may imply changing age-specific birth rates. In this case, (1) and (3) are not the same.

Ní Bhrolcháin (2007, 2008) proposed a different categorization of purposes for measuring period fertility. Five reasons for measuring period fertility are distinguished: to describe fertility time trends, to explain these, to anticipate future population prospects, to provide input parameters for formal models and to communicate with non-specialist audiences. Like Guillot, Ní Bhrolcháin argues that tempo adjusted measures may be appropriate for some uses but not others. For our purpose we prefer Guillot's categorization because it is simpler and specifically intended for demographic processes other than fertility.

On the first use, we agree with Guillot (2006) that period indicators as simple summary measures are not distorted even when tempo effects exist, provided there is no intention to measure true cohort or period conditions. This measure might be useful for assessing the effects of trends in period fertility or mortality on population growth because the inflation or deflation of events caused by tempo effects have a direct impact on population dynamics. A tempo adjustment is therefore not necessary.

On the second use we agree again with Guillot (2006) that tempo effects in period measures clearly result in a distorted estimates of cohort measures. This indicates a need for tempo adjustment, but this approach should be used only rarely and with great care because cohort indicators cannot, in general, be derived from a single period indicator. This derivation is possible only if period and cohort quantum are constant while tempo changes linearly with shifting of the schedule of age-specific rates. These conclusions regarding the first two uses are straightforward.

Our primary concern here is therefore with the third use of period indicators- as measures of current conditions. The underlying factors that are relevant depend on the demographic event. Conditions affecting fertility, for example, include level of education, women's labor force participation, availability of childcare facilities, government incentives and disincentives for childbearing, gender equality, cultural norms, etc. Conditions affecting mortality include the availability and use of immunizations and other public health practices; the availability of medical devices, such as pace-makers, and dialysis machines; screening programs for early detection of different kinds of cancers; drugs that prolong life by reducing the incidence of particular life-threatening diseases, such as antibiotics and anti-cholesterol drugs; and surgical procedures, such as open heart surgery, that prolong life; and behavioral changes, such as changes in diet and exercise, that improve health and prolong life.

Our intention is to obtain an estimate of the period indicator that would have been observed if current conditions had not changed during year *t*. This estimate equals the value of the indicator that would be observed in the hypothetical cohort born in year *t* if no further changes in conditions (including the mean age at the event) occur after year *t* (see Vaupel and Guillot, 2006 for similar definitions). The terms "current conditions" and "constant conditions" are very closely related because what "constant conditions" means depends on how "current conditions" is defined.

There are two perspectives on how to estimate such period indicators (Bongaarts and Feeney, 2008; Guillot, 2006; Rodriguez, 2006; Vaupel, 2002). In the classical approach period conditions are assumed to generate a set of age specific rates that faithfully reflect these conditions. As conditions change event rates rise or fall and any resulting changes in tempo are considered secondary. According to this model, period indicators derived from rates are undistorted indicators of period conditions. Tempo effects exist, but they are not distortions and there is no need for a tempo adjustment. This approach captures an essential aspect of the occurrence of demographic events, but Ryder's analysis of fertility tempo effects shows that they fail to capture another important aspect: the delays that give rise to the fertility tempo effects that most demographers now regard as distortions.

In contrast, the Bongaarts-Feeney (BF) approach to general tempo effects distinguishes between changes in age-specific rates that reflect changes in quantum and changes in age-specific rates that reflect tempo change. Tempo changes result in *shifts* of the event rate schedules to either higher or lower ages, as conditions change. This implies that *constant* conditions may be associated with *changing* rates.

To illustrate, suppose that socioeconomic conditions influencing age at childbearing are changing before year *t* ( e.g., a rise in women's level of education or labor force participation) and that this leads to a tempo change, i.e., a rise in the mean age at birth. Assume further that the (undistorted) quantum of fertility is constant. The shifting age schedule of childbearing results in a tempo effect that depresses the total fertility rate up to time *t*.

Next, suppose that the socioeconomic conditions are held constant from time *t* onward and that the new, higher age at birth prevails as long as the new socioeconomic conditions continue. Continuation of current *conditions* after time *t* means a movement from a rising to a constant mean age at birth. The disappearance of the tempo effect after *t* leads to a rise in age-specific birth rates and in the total fertility rate even though the quantum of fertility is assumed to be unaffected by the changing conditions. Constant conditions here does *not* mean a continuation of the corresponding age-specific birth rates. On the contrary, constant conditions *imply a change* in age-specific rates.

In this illustration, the *TFR* before *t* is clearly affected by a tempo effect and this effect is a distortion because the *TFR* does not measure the fertility quantum implied by the prevailing conditions up to time *t*. This view of tempo effects and distortions is now widely accepted and corrections for tempo distortions are now commonly used in the analysis of fertility trends (Lutz and Sobotka, 2008).

We argue that the same phenomenon applies to adult mortality in high life expectancy countries. Changes in mortality conditions lead to *shifts* of the schedule of mortality rates to higher ages, strictly reflecting tempo change (the quantum of mortality can of course not change because everyone dies eventually). Just as in the case of childbearing, when new conditions (e.g. new drugs, or surgical procedures) come into being to raise the age at death, they tend to persist because they shift the age schedule of rates to higher ages. Constant conditions thus refers to the absence of changes in the many medical, public health, behavioral and other circumstances that influence length of life. Just as in the case of fertility, constant conditions may imply changes in age-specific rates. "Conditions" refers to mortality-relevant behaviors, treatments and technologies – not to age-specific rates as such.

We therefore argue that period indicators of mortality are subject to tempo effects which are distortions for the same reasons that period indicators of fertility are subject to these effects and contain distortions. Current rates do not necessarily reflect current conditions. When conditions change in such a way as to increase length of life by shifting rates to higher ages, tempo effects come into play, and age-specific death rates are distorted indicators of current conditions. This is the rationale for the mortality tempo adjustment we have proposed. Whenever current conditions are changing, mortality measures derived from period age-specific death rates need to be adjusted for tempo distortions.

Our proposed tempo adjustment to fertility is widely used and accepted, but the idea of a tempo adjusted life expectancy remains controversial and difficult to understand. To clarify the latter it is helpful to consider two examples of the relationship between cohort and adjusted period life expectancy under simplifying assumptions. First, if the mortality schedule shifts along the age axis at a constant rate so that the mean age at death of the cohort changes linearly over time then the tempo adjusted life expectancy $e^*(t)$ equals the life expectancy of the cohort born $e^*(t)$ years ago (Goldstein, 2006; Rodriquez, 2006). For example if $e^*$=75 years in the year 2000 then the life expectancy of the cohort born in 1925 (which reaches its peak death rates around the year 2000) equals 75 years. Empirical confirmation of this relationship is provided by Bongaarts and Feeney (2006) and Guillot and Kim (2010) for contemporary populations in which the linear shift assumption holds approximately for adults.

Second, as shown in an appendix, the tempo adjusted life expectancy at time $t$ equals the (weighted) average of the life expectancies of cohorts born in the past provided the weights constitute a probability density function with mean $e^*(t)$. Since cohort and period life expectancies have risen in recent decades in contemporary populations the tempo adjusted life expectancy is lower that the observed conventional life expectancy at time $t$. For example Bongaarts and Feeney (2003) find that the tempo distortions for France, Sweden and USA equal 2.4, 1.6 and 1.6 years respectively for the period 1980-1995. Further discussion of these relationships are provided in Luy and Wegner (2009) who also provide additional estimates of tempo effects for 2001-2005 for females for 41 countries with Japan having the largest distortion (3.0) years.

## 4. DISCUSSION

(1) The preceding discussion focuses on indicators derived from rates of the "second kind", but indicators derived from rates of the "first kind" (hazards) are also subject to tempo effects and distortions. This is because tempo changes operate on the numerators of rates, and hence affect rates of both the first and second kind. In particular, life table measures are subject to tempo effects (see Bongaarts and Feeney, 2006; Kohler and Ortega, 2002, 2004).

(2) Tempo effects can be measured, and appropriate corrections can be

made, only by imposing simplifying assumptions. The most important of these in our analysis is that there are no cohort effects, i.e. all cohorts respond to changing period conditions in the same way. We have argued elsewhere that this assumption holds approximately for fertility and adult mortality in many contemporary populations. When this simplifying assumption does not hold, the *TER* is affected by distortions other than tempo effects, e.g. a changing parity distribution in the case of fertility. In such cases tempo effects still exist, but the measurement of quantum and tempo becomes difficult. The conventional BF method for estimating tempo effects is then not accurate and it does not address these other distortions. Further methodological research is needed to develop general methods for estimating period quantum and tempo. A discussion of this topic is beyond the scope of this paper but it should be noted that methods based on rates of the first kind have the advantage of not being affected by the parity distribution (Bongaarts and Feeney, 2006; Kohler and Ortega, 2002, 2004).

(3) The BF correction for tempo distortions does not apply to all ages in the case of mortality. As discussed by Bongaarts and Feeney (2008) the classical interpretation of rates applies to child mortality and to accidental or crisis mortality, while the BF interpretation of tempo effects applies to senescent mortality.

(4) Tempo adjusted period indicators are not projections of future levels any more than unadjusted period indicators are projections of future levels. In making projections, however, it is useful to be able to assess tempo effects. Consider for example a projection of the *TFR* in a population with a rising mean age at childbearing, which creates a negative tempo effect. Forecasting the quantum and tempo components separately may be more appropriate than forecasting unadjusted total fertility rates directly. One might assume that both components remain constant, implying a continuation of the trend in the mean age at birth.

Alternatively, if the mean age is already high, it might be assumed that the mean age will rise more slowly in the future and will eventually stop rising. Under this scenario the tempo distortion would disappear over time thus putting upward pressure in the *TFR*. It would then be reasonable to assume a small rise in period fertility if the quantum remains the same. This is why the UN medium variant projections for developed countries assume a rise in the *TFR* over coming decades (UN, 2007).

(5) The existence of tempo effects/distortions in period tempo indicators (i.e. in the mean age of events) is a complex issue. Here we note only that the above conclusions regarding the presence of tempo effects and distortions in quantum indicators also applies to tempo indicators derived from rates of the first kind. This follows from the fact that tempo indicators are derived from the same age specific rates used to calculate quantum indicators. For example, Bongaarts and Feeney (2006) demonstrate tempo effects in the period mean ages at first birth derived with life tables when the timing of childbearing is changing. Tempo indicators of mortality such as the life expectancy at birth also contain tempo effects, and these effects are distortions under conditions discussed above.

Evidence in support of this assertion includes the observation that the conventionally calculated life expectancy at a given time differs from the mean age at death of the cohort which is at its mean age at death at this time (Bongaarts and Feeney, 2006, Rodriguez, 2006; Goldstein, 2006). The magnitude of this difference varies with the rate of increase in the mean age at death, as one would expect if the difference is caused by a tempo effect.

(6) Our analysis takes age as the underlying dimension of period measures. However, a number of measures are constructed on the basis of duration since some previous event of interest: divorce rates can be computed by time elapsed since marriage, second birth rate can be computed by duration since first birth, etc. These duration-based measures can also be distorted by tempo effects when the mean of the interval between the events in question (marriage and first birth, first birth and second birth, and so on) changes over time.

## 5. CONCLUSION

The discussion of tempo distortions is of wide interest because they can result in erroneous analysis and interpretation of past levels and trends in the quantum and tempo of life-cycle events. This in turn may result in inappropriate projections or the adoption of sub-optimal policies.

Our intent here is to clarify the terms “tempo effect” and “tempo distortion” and to identify the uses of quantum and tempo measures that in our view lead to distortions. We conclude that tempo effects are widespread in period measures. We agree that there are certain situations in which tempo effects are not distortions and the observed indicators need not be adjusted. This is the case in particular when measures are used for purely descriptive purposes and/or to assess the direct mechanical impact of trends in fertility or mortality on population growth and the age structure. However, when the objective is to measure current conditions, as is usually the case, our view differs from the classical approach, in which current event rates are assumed to reflect fully current conditions. We instead argue that current age-specific event rates do not in general reflect current conditions. When changes in the timing of demographic events shift rates schedules to higher or lower ages, constant conditions may imply changing rates. Age-specific rates fully describe conditions only when such shifts to higher or lower ages are absent.

When shifts to higher or lower ages are present, tempo distortions exist and need to be corrected, in order to get an accurate measurement of current conditions. These shifts are not limited to fertility. They may occur for nuptiality, mortality, and other events as well. We believe that our approach is a more realistic reflection of actual behavior in contemporary populations. Further research is needed to confirm this view.

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## Appendix

Let the life expectancy at birth of the cohort born in year t be denoted as $e^c(t)$ and the tempo adjusted life expectancy in year *t* as $e^*(t)$. Goldstein (2006) and Rodriguez (2006) have demonstrated that the life expectancy of the cohort born $e^*(t)$ years before year t equals the tempo adjusted life expectancy in year *t* :

$$e^c(t - e^*(t)) = e^*(t) \qquad [1]$$

provided the cohort mortality schedules shift linearly with fixed annual increments *r*, so that

$$e^c(t - x) = e^c(t) - rx \qquad [2]$$

We want to extend this result by showing that the weighted average of lagged cohort life expectancy also equals $e^*(t)$.i.e.

$$\int_0^\infty w(x,t)e^c(t - x)dt = e^*(t) \qquad [3]$$

The proof given in equation (5) below shows that this equation holds provided the weights *w*(*x,t*) constitute a proper probability density function with mean $e^*(t)$:

$$\int_0^\infty x\, w(x,t)dx = e^*(t) \qquad [4]$$

An example of $w(x,t)$ that meets this condition is the distribution of cohort deaths in year $t$ with deaths estimated from cohort life tables (Bongaarts and Feeney, 2003)

Substitution of [2] in [3] yields

$$\int_0^\infty w(x,t)(e^c(t) - rx)dx =$$

$$\int_0^\infty w(x,t)e^c(t)dx - r\int_0^\infty x\, w(x,t)dx =$$

$$e^c(t) - re^*(t) = e^c\big(t - e^*(t)\big) = e^*(t) \qquad [5]$$