A Tentative Model for the Volume of Trade Between Countries[1]

By

Pentti Pöyhönen

Contents: I. A General Model for the Structure of the International Exchange of Goods. — II. Finding the Form of the Explanatory Model. — III. The Data Used. — IV. Estimation of the Parameters of the Model. — V. On the Values of the Parameters. — VI. Discussion of the Results.

This paper aims to bring some structural features of international trade into relief. The analysis is based on the simultaneous application of a structural and explanatory model resembling an input-output model to the exchange of goods between ten European countries in 1958. The nature of the experiment made it necessary to confine the analysis at this stage to a relatively limited set of data, but there is nothing, in principle, to prevent the extension of the analysis to the world trade as a whole. Changes over time were also left outside the scope of this paper, so that the model employed is static. In spite of these restrictions the results are of interest and, as I believe, novel; they serve to indicate that the bilateral approach can be abandoned in investigating international trade, which can be treated as an integral whole within the framework of a single model or model system[2].

Note of the Editorial Department: We received the present paper already on the 3rd of November 1961. Because of lack of space it could be published only just now.

[1] The same problem was dealt with, using precisely the same model, by Jan Tinbergen in Appendix VI of "Shaping the World Economy," of which the Foreword was dated by the publisher in July, 1962. Thus, the one and same theory seems to have been elaborated simultaneously but independently at two different research centres. Although Tinbergen's work appeared prior to this article, it is being published here in unaltered form.

[2] It is my agreeable duty to mention the valuable criticism and support I have received from my colleague, Professor Dr. Leo Törnqvist, both at the development and the estimation stage of the structural model.

I. A General Model for the Structure of the International Exchange of Goods

The starting point of the analysis is the square matrix A of the exchange of goods:

$$(1)\quad A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{in} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nj} & \cdots & a_{nn} \end{pmatrix}$$

where the rows i indicate the country of export and the columns j the country of import, and the elements a_{ij} $(i \neq j)$ represent exports. The elements a_{ij} $(i = j)$ of the main diagonal, *i. e.*, internal trade, are left outside the scope of the present study; obviously, however, they could be included if desired.

In addition, a number of data on international trade and the countries concerned are assumed to be known. These data allow of a corresponding representation in the form of vectors or matrices B, C, ...

Let us then form the matrix function

$$(2)\qquad F = F(A; A)$$

and, with the aid of the available information, an explanatory model

$$(3)\qquad F' = F(A'(B, C, \ldots; \alpha, \beta, \ldots); A)$$

where A′ is the sought for explanation in a matrix form, α, β, ... being the requisite parameters.

Consider the explanatory error matrix $E = \varepsilon_{ij} = F_{ij} - F'_{ij}$ of model (3), imposing on it the requirement

$$(4)\qquad \sum_{i,j} \varepsilon_{ij}^2 = \min \ (i \neq j)$$

which implies the use of the least-squares method in the estimation of the parameters α, β, ... after the transformation F.

II. Finding the Form of the Explanatory Model

The simplest possible model would result from an exclusive use of matrix A and a number of parameters, so that the explanatory function F′ would be purely structural as is the case with the input-output analysis[1]. The amount of additional information is relatively small in comparison with the information contained in the original matrix. Moreover, the

[1] Polak's point of departure was, in principle, a case as simple as this, even though the empirical treatment was not simultaneous. J. J. Polak, *An International Economic System*, Chicago, 1953.

analysis should be carried out within the framework of the world trade as a whole, for otherwise the results might be misleading.

An explanatory model F′ in the true sense of the word can be spoken of only when information in the form of other matrices is utilized in addition to the basic matrix A. The case where there are two explanatory matrices will be taken up for consideration in the following. The two matrices are the diagonal matrix E of national income:

$$(5) \qquad E = \begin{pmatrix} e_{11} & 0 & \cdots & 0 \\ 0 & e_{22} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & e_{nn} \end{pmatrix}$$

and the matrix R of the distances of transportation:

$$(6) \qquad R = \begin{pmatrix} 0 & r_{12} & \cdots & r_{1n} \\ r_{21} & 0 & \cdots & r_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ r_{n1} & r_{n2} & \cdots & 0 \end{pmatrix}$$

Of the elements of E and R it is possible to construct a variety of explanatory models. The following, quite general form was tried out in the applications:

$$(7) \qquad a'_{ij} = c\, c_i\, c_j \frac{e_{ii}^{\alpha}\, e_{jj}^{\beta}}{(1 + \gamma r_{ij})^{\delta}} \qquad (i \neq j)$$

where a'_{ij} = estimate of the value of exports from country i to country j
e_{ii} = national income of the country of export i
e_{jj} = national income of the country of import j
r_{ij} = distance of transportation
α, β = national-income elasticities of exports and imports
γ = transportation cost coefficient per nautical mile
δ = isolation parameter
c_i = export parameter of the country of export
c_j = import parameter of the country of import
c = a constant

Here the F-matrix was formed by taking natural logarithms of the elements of the A-matrix.

III. The Data Used

Data for the tentative analysis were drawn from the statistical publications of the United Nations[1], and they pertained to ten European

[1] The sources are indicated in the tables.

Table 1 — *Exports (f.o.b.)* a_{ij}, $i \neq j$, *and National Income* a_{ii} *of Selected European Countries* in 1958 (Mill. $)[1]

Country of export, i	Country of import, j									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(1) Belgium . . .	11566	49.7	20.4	352.1	69.0	630.3	38.8	34.1	80.9	173.8
(2) Denmark . . .	15.5	4975	16.3	249.5	66.0	27.2	59.8	3.5	91.5	322.7
(3) Finland . . .	28.2	22.7	3695	84.0	13.9	33.2	5.1	0.5	25.2	167.5
(4) Western Germany . . .	584.5	264.5	115.8	54619	441.2	712.9	254.3	83.6	539.6	347.9
(5) Italy	57.5	19.4	9.6	362.3	26768	51.9	20.8	17.0	68.3	173.2
(6) Netherlands . .	481.7	84.7	27.0	610.7	88.1	9542	73.7	13.4	146.2	382.9
(7) Norway . . .	21.3	45.2	8.7	104.6	19.4	27.5	3970	4.1	72.8	143.9
(8) Portugal . . .	10.6	3.6	0.3	22.3	12.3	7.1	2.2	2054	6.7	32.7
(9) Sweden	89.2	123.5	59.0	296.9	66.9	101.8	219.2	12.6	10621	341.2
(10) United Kingdom . . .	167.9	215.0	89.2	345.2	185.7	274.0	183.0	63.6	292.0	64028

Table 2 — *Distances of Transportation Between the Principal Port of the Country of Export* i *and the Principal Port of the Country of Import* j (1,000 nautical miles)[2]

i \ j	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(1)	0	0.61	1.02	0.39	2.24	0.10	0.72	0.96	0.70	0.14
(2)	0.61	0	0.57	0.29	2.86	0.50	0.27	1.58	0.14	0.40
(3)	1.02	0.57	0	0.71	3.35	1.02	0.83	1.99	0.24	1.02
(4)	0.39	0.29	0.71	0	2.64	0.32	0.44	1.34	0.42	0.36
(5)	2.24	2.86	3.35	2.64	0	2.32	3.08	1.28	3.06	2.12
(6)	0.10	0.50	1.02	0.32	2.32	0	0.65	1.02	0.63	0.15
(7)	0.72	0.27	0.83	0.44	3.08	0.65	0	1.66	0.17	0.64
(8)	0.96	1.58	1.99	1.34	1.28	1.02	1.66	0	1.64	1.04
(9)	0.70	0.14	0.24	0.42	3.06	0.63	0.17	1.64	0	0.50
(10)	0.14	0.40	1.02	0.36	2.12	0.15	0.64	1.04	0.50	0

countries. The following points were observed in selecting the countries:

(1) Inclusion of the most important European countries in the analysis.

(2) Similarity in the principal modes of transportation. A predominance of transportation by sea was striven for. This meant the imposition of considerable restrictions on countries eligible according to the first principle, for France, Switzerland and Austria,

[1] *Source:* Statistical Office of the United Nations, Department of Economic and Social Affairs, *Yearbook of International Trade Statistics*, New York, 1959, Vol. I and II. — *Idem, Yearbook of National Accounts Statistics*, 1960, P. D.

[2] *Source: Otavan maailmankartasto* (Otava's World Atlas), Keuruu, 1957.

for example, had to be excluded. On the other hand, a wholly consistent application of the second principle would have excluded all but a few countries.

In view of the tentative nature of the analysis, a single time period, the year 1958, was regarded as sufficient. The choice is open to the criticism, for instance, that this year was one of depression.

The research material is presented in Tables 1 and 2.

IV. Estimation of the Parameters of the Model

The Model (7) thus constructed is non-linear, even though it differs only slightly from logarithmic linearity[1]. In order to determine the values of the parameters γ and δ simultaneously, electric data processing was resorted to. To save expenses, the values of c_i and c_j were determined afterwards with the aid of the logarithmic estimation error ε_{ij} and in model (7) only the common constant c was estimated. The analysis yielded the following estimates[2]:

$$\alpha = 0.518$$
$$\beta = 0.504$$
$$\gamma = 0.00157$$
$$\delta = 1.817$$
$$\ln c = -3.818$$

The values of $\ln c_i$ and $\ln c_j$ ascomputed from the matrix of estimation errors were, by countries, as follows[3]:

	$\ln c_i$	$\ln c_j$
(1) Belgium	0.109	—0.145
(2) Denmark	—0.190	—0.081
(3) Finland	—0.481	—0.580
(4) Western Germany	0.624	0.200
(5) Italy	0.583	0.838
(6) Netherlands	0.434	0.061
(7) Norway	—0.315	0.002
(8) Portugal	—1.016	—0.402
(9) Sweden	0.302	0.062
(10) United Kingdom	—0.024	0.071

[1] The logarithmically linear form of the expression of transportation distance gave considerably weaker results.

[2] The analysis was carried out at the Electronics Department of Finnish Cable Co by an Elliot 803 data processing machine. The starting points employed were $\alpha = \beta = 0.5$; $\gamma = 0.01$; $\delta = 2$; $\ln c = -4$. The estimation principle applied was the minimization of the logarithmic residual sum of squares. The best procedure of estimation may be found such as $F—F' = a_{ij} \ln a_{ij} — a_{ij} \ln a'_{ij}$.

[3] This estimation procedure presupposes that $a_{ij} \neq 0$, which does not hold true in general.

The total correlation between the explanation augmented in this way and the basic matrix A was 0.94. The residual error matrix is presented in Table 3.

Table 3 — *Residual* ln *Errors of Model* (7)

i \ j	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(1)	.	—0.18	0.06	—0.03	—0.18	1.04	—0.25	0.63	—0.10	—1.14
(2)	—0.98	.	0.03	0.20	0.85	—0.67	0.25	—0.34	—0.13	0.69
(3)	0.55	0.15	.	0.16	—0.03	0.61	—0.97	—1.56	—0.78	1.26
(4)	0.27	—0.31	0.12	.	0.59	0.26	—0.08	0.57	0.09	—1.35
(5)	0.18	—0.37	—0.08	1.08	.	—0.03	—0.04	—0.67	0.58	0.09
(6)	0.65	—0.03	0.11	0.18	—0.11	.	0.07	—0.47	0.18	—0.55
(7)	—0.27	0.18	—0.02	—0.19	—0.02	—0.21	.	0.12	—0.07	0.47
(8)	0.55	0.19	—1.42	0.34	—0.60	—0.09	—0.22	.	0.34	0.49
(9)	0.01	—0.19	0.09	—0.30	0.08	—0.05	0.46	0.09	.	0.01
(10)	—0.86	0.23	0.78	—0.85	—0.02	—0.44	0.44	0.58	0.16	.

V. On the Values of the Parameters

The values of the parameters α, β, γ and δ were in fairly good accord with the starting points in data processing. The explanatory function proved to be symmetric with respect to the country of export and the country of import; this manifests itself in the fact that α and β were equal to one another and reflects bilaterality in the exchange of goods in general. Furthermore, $\alpha + \beta \approx 1$, which is indicative of the static nature of the model.

The value of γ is relatively small, so that the effect of the constant in the denominator of (7) is quite considerable. In the first place, this reflects the relationship between the cost of loading and that of transportation.

The parameter δ turned out to be somewhat smaller than expected. Since γ and δ are closely connected with one another in the estimation, their product is likely to be capable of a more reliable estimation than is either of them separately.

From the values of c_i and c_j it is possible to draw some interesting conclusions on the trade between the countries included in the analysis. The first is the relative strength of exports in the case of the Netherlands, Western Germany and Italy[1]. Norway, Finland and, especially, Portugal rank lowest on the scale. Regarding imports, Italy occupies relatively the strongest position, the second place being held by Western Germany.

[1] In the case of the trade between Western Germany and Italy, for instance, the shortcoming resulting from the exclusive use of seatransportation distances becomes quite obvious.

VI. Discussion of the Results

The close connection between our estimator function and the gravitational problem of two bodies may have drawn the careful reader's attention even at the stage when model (7) was described. In point of fact, the analogy is rather formal in nature, for the masses are replaced by the square roots of the national incomes and a cost of transport function is substituted for the distance. I do not want to deny the fact, however, that the starting point of the parameter δ, for example, has its roots in reasoning based on such an analogy.

Regional studies have frequently shown that economic activity is governed by rules of which the analogies are to be found in natural sciences. Determination of the boundaries of economic areas; analysis of the prices of properties, as well as that of the price formation in the neighbourhood of economic centres in general; and many other allied studies have yielded empirical results well compatible with applications of natural sciences, analogous to these.

As I see it, the present tentative analysis of international trade[1] undoubtedly reveals the existence of such structural characteristics as are bound essentially to affect our conceptions of the causes of the geographical distribution of export trade. In other words, the effect of trade policy can be estimated only by isolating the factors, one after the other, that affect this distribution, without ascribing the observed differences in their entirety to any single factor.

* * *

Zusammenfassung: Ein Versuchsmodell für das Volumen des zwischenstaatlichen Handels. — Für den internationalen Austausch von Gütern kann ein Modell konstruiert werden, das dem in der Input-Output-Analyse verwendeten ähnlich ist. In der Probeuntersuchung, über die in diesem Beitrag berichtet wird, bildet der Handel zwischen zehn europäischen Ländern im Jahr 1958 den Gegenstand einer Simultananalyse. Das Modell, das erprobt wurde, enthält den Wert der Ausfuhr als zu erklärende Variable, während die erklärenden Variablen das Volkseinkommen der Aus- und Einfuhrländer und eine Kostenfunktion, die die Beförderungsdistanz einschließt, sind. In dem Modell gibt es vier Parameter, die allen Ländern gemeinsam sind und die die Strukturmerkmale widerspiegeln, die den Austausch von Gütern beeinflussen, sowie einen Export- und einen Importparameter, die für jedes besondere Land typisch sind. Es kann aus diesen Parametern geschlossen werden, daß es im internationalen Handel eine große Zahl von Regelmäßigkeiten gibt, die von den Wirkungen der eingeschlagenen Handelspolitik deutlich unterschieden werden können. Hier ergeben sich, ebenso wie in früheren Regionalstudien, hinsichtlich der gefundenen Beziehungen die engsten Analogien zum Bereich der Naturwissenschaften.

*

[1] On the basis of this tentative study, the construction of a more elaborate, dynamic model is under way.

Résumé: Un modèle d'essai pour le volume du commerce international. — On peut construire, pour l'échange international de marchandises, un modèle semblable à celui dont on se sert pour l'analyse input-output. Cet article rend compte d'une étude préliminaire de l'analyse simultanée du commerce entre dix pays européens en l'année 1958. Le modèle qui fut mis à l'épreuve contient la valeur des exportations comme variable à expliquer, tandis que les variables explicatives sont: les revenus nationaux des pays exportateurs et importateurs et une fonction des coûts, comprenant la distance des transportations. Le modèle contient quatre paramètres, qui sont communs à tous les pays et qui reflètent les caractéristiques de structure qui affectent l'échange de marchandises. Puis, il contient un paramètre d'exportation et un paramètre d'importation, qui sont typiques de chacun des pays individuellement. On peut conclure de ces paramètres qu'il y a dans le commerce une bonne mesure de régularité, qu'on peut très bien distinguer des effets d'une politique commerciale adoptée. Comme dans beaucoup d'études régionales antérieures, on se rend compte que les relations qu'on découvre ici sont fortement analogues à celles qu'on connaît en sciences naturelles.

*

Resumen: Un modelo de ensayo para el volumen del comercio internacional. — Para el intercambio internacional de bienes se puede construir un modelo que tiene semejanza con el usado en el análisis input-output. En el estudio de ensayo, sobre el cual se informa en este artículo, el comercio entre diez países europeos en el año de 1958 es el objeto de un análisis simultáneo. El modelo sometido a prueba contiene el valor de la exportación como la variable de explicar, mientras que las variables explicativas son la renta nacional de los países exportadores e importadores así como una función de costes que incluye la distancia del transporte. En el modelo hay cuatro parámetros comunes a todos los países que reflejan los característicos estructurales influyentes en el intercambio de bienes así como un parámetro exportador y un parámetro importador que son típicos para cada uno de los países. Puede inferirse de estos parámetros que en el comercio internacional hay gran número de regularidades que pueden ser distinguidas de los efectos de la política comercial seguida. Aquí, justamente como en muchos estudios regionales previos, respecto a las relaciones averiguadas pueden hallarse las analogías más estrechas al dominio de las ciencias naturales.

*

Riassunto: Un modello di prova per il volume del commercio internazionale. — Per lo scambio internazionale di beni si può costruire un modello che ha somiglianza con quello usato nell'analisi input-output. Nello studio di prova del quale si informa in questo articolo, il commercio fra dieci paesi europei nel 1958 è il soggetto di un'analisi simultanea. Il modello sottomesso alla prova contiene il valore dell'esportazione come la variabile da esplicare, mentre che le variabili esplicative sono il reddito nazionale dei paesi esportatori e importatori com'anche una funzione di costi che inchiude la distanza di trasporto. Nel modello vi sono quattro parametri comuni a tutti i paesi che riflettono le caratteristiche strutturali aventi influenza sullo scambio di beni com'anche un parametro esportatore e un parametro importatore che sono tipici per ciascuno dei paesi. Si può inferire da questi parametri che nel commercio internazionale vi è gran numero di regolarità le quali possono essere distinte dagli effetti della politica commerciale perseguita. Qui, tal quale in molti studi regionali precedenti, riguardo alle relazioni scoperte possono essere trovate le più strette analogie alla sfera delle scienze naturali.